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ON THE EXISTENCE OF PERIODIC SOLUTIONS IN THE NONLINEAR THEORY OF OSCILLATIONS OF SHALLOW SHELLS, TAKING DAMPING INTO ACCOUNT

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I. I. VOROVICH and S. A. SOLOP
(Rostov-on-Don)
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Problems of existence of periodic solutions for various nonlinear equations of the continuous media mechanics are investigated in a number of papers, e.g. in [1, 2]. The present paper proves the existence of an ω-periodic solution for nonlinear equations of anisotropic inhomogeneous shallow shells of variable thickness, with damping taken into account.

1. Basic relationships. Let the median surface of the shell S be defined by the equation $\mathbf{r} = \mathbf{r}(\alpha_1, \alpha_2)$ which maps S homeomorphically onto the domain Ω of variables α_1, α_2 with the boundary Γ. We consider the following variant of the nonlinear theory for an elastic anisotropic inhomogeneous shallow shell of variable thickness:

$$\varepsilon_{11} = e_{11} + k_{11}u_1 + {}^1/_2\psi_1^2 = A_1^{-1}u_{1\alpha_1} + A_{1\alpha_2}(A_1A_2)^{-1}u_2 + k_{11}u_3 + {}^1/_2\psi_1^2$$

$$2\varepsilon_{12} = 2e_{12} + 2k_{12}u_3 + \psi_1\psi_2 = A_1A_2^{-1}(u_1A_1^{-1})_{\alpha_2} +$$

$$A_2A_1^{-1}(u_2A_2^{-1})_{\alpha_1} + 2k_{12}u_3 + \psi_1\psi_2$$

$$2\varkappa_{12} = -A_1A_2^{-1}(\psi_1A_1^{-1})_{\alpha_2} - A_1^{-1}A_2(\psi_2A_2^{-1})_{\alpha_1}$$

$$\varkappa_{11} = -A_1^{-1}\psi_{1\alpha_1} - A_{1\alpha_2}\psi_2(A_1A_2)^{-1}, \quad \psi_1 = A_1^{-1}u_{3\alpha_1} \quad (1 \rightleftarrows 2)$$

$$T_{ij} = E_{ijkl}\varepsilon_{kl}, \; M_{ij} = D_{ijkl}\varkappa_{kl}, \; D_{ijkl} = {}^1/_3h^2E_{ijkl}, \; E_{ijkl} = E_{klij} = E_{jikl}$$

where the notation used is that of [3, 4].

The differential equations of oscillations of the shell with damping taken into account can be written in the form

$$\mathbf{u}_{tt} + \varepsilon \mathbf{u}_t + \mathbf{A}\mathbf{u} + \mathbf{B}\mathbf{u} = \mathbf{F} \tag{1.1}$$

where $\varepsilon > 0$ is a constant, $\mathbf{F}$ is a known vector function of time, and the subscript t denotes the derivative with respect to t. Detailed expressions for the linear $\mathbf{Au}$ and the nonlinear $\mathbf{Bu}$ part of (1.1) not depending explicitly on t, are given in [5].

Let the shell be acted upon by the body forces $\mathbf{F}$, ω-periodic with respect to time. We formulate the following problem: to find a vector $\mathbf{u}(\alpha_1, \alpha_2, t) = (u_1, u_2, u_3)$, $(\alpha_1, \alpha_2 \in \Omega, \ -\infty < t < +\infty)$ satisfying Eqs. (1.1) and the conditions

$$u_1 |_\Gamma = u_2 |_\Gamma = 0 \tag{1.2}$$

$$u_3 |_\Gamma = \frac{\partial u_3}{\partial n}\Big|_\Gamma = 0 \tag{1.3}$$

$$\mathbf{u}(t+\omega) = \mathbf{u}(t), \quad \mathbf{u}_t(t+\omega) = \mathbf{u}_t(t) \tag{1.4}$$

2. Fundamental assumptions. Let the following conditions be satisfied:

1) Ω is a finite sum of bounded star domains,

2) Γ is a contour of the Liapunov class $\Lambda_1(m, 0)$,

3) the coefficients of the first quadratic form of the surface S are $A_i \in L^\infty(\Omega)$, and their derivatives are $A_{i\alpha_j} \in L^\infty(\Omega)$, $i, j = 1, 2$,

4) the curvatures of the median surface $k_{ij} \in L^2(\Omega)^\infty$,

5) the shell thickness $2h(\alpha_1, \alpha_2) \in L^\infty(\Omega)_\infty$,

6) the elastic characteristics $E_{ijkl}(\alpha_1, \alpha_2) \in L^\infty(\Omega)$ and the inequality

$$m_1 e_{ij}^\circ e_{ij}^\circ \leqslant E_{ijkl} e_{ij}^\circ e_{kl}^\circ \leqslant m_2 e_{ij}^\circ e_{ij}^\circ$$

holds for all symmetric tensors e_{ij}° almost everywhere,

7) the functions A_i, h and E_{ijkl} are bounded from below by positive constants.

The basic spaces.

Space $H_1(\Omega)$. The space $H_1(\Omega)$ is a Hilbert space obtained by the closure of the set C_1 of functions $u_3 \in C^{(2)}(\Omega)$ satisfying the conditions (1.3) and (1.4) in the norm corresponding to the scalar product

$$(u_3^{(1)} \cdot u_3^{(2)})_{H_1(\Omega)} = \int_\Omega D_{ijkl} \varkappa_{kl}^{(1)} \varkappa_{ij}^{(2)} d\Omega, \quad d\Omega = A_1 A_2 d\alpha_1 d\alpha_2$$

Space $H_2(\Omega)$. The space $H_2(\Omega)$ is a Hilbert space obtained by the closure of the set C_2 of pairs of functions $\mathbf{u}^*(u_1, u_2) \in C^{(1)}(\Omega)$ satisfying the conditions (1.2) and (1.4) in the norm corresponding to the scalar product

$$(\mathbf{u}^{*(1)} \cdot \mathbf{u}^{*(2)})_{H_2(\Omega)} = \int_\Omega E_{ijkl} e_{kl}^{(1)} e_{ij}^{(2)} d\Omega$$

Space $H_3(\Omega)$. The space $H_3(\Omega)$ is a Hilbert space of the vector functions $\mathbf{u}(u_1, u_2 \ u_3)$ such that $u_3 \in H_1(\Omega)$ and $\mathbf{u}^* \in H_2(\Omega)$ with the naturally introduced scalar product $H_3 = H_1 \times H_2$.

Space X_1. The space X_1 is a Hilbert space obtained by the closure of the set $C_1 \times C_2$ in the norm corresponding to the scalar product

$$(\mathbf{u}^{(1)} \cdot \mathbf{u}^{(2)})_1 = \int_\Omega 2\rho h \left(u_1^{(1)} u_2^{(2)} + u_2^{(1)} u_2^{(2)} + u_3^{(1)} u_3^{(2)}\right) d\Omega$$

where $\rho\ (\alpha_1, \alpha_2) > 0$ is the shell material density.

Space $X_2\ (0, \omega)$. Let $E_1 = C_1 \times C_2$ and E_2 be a set of elements $\mathbf{u}\ (t)$ depending on the parameter t and such, that $\mathbf{u} \in E_1$ and $\mathbf{u}_t \in X_1$ for any $-\infty < t < +\infty$, with the finite norms

$$\max_t \|\mathbf{u}\|_1, \quad \max_t \|\mathbf{u}_t\|_1, \quad \int_0^\omega \|\mathbf{u}\|^2_{H_3(\Omega)}\, dt$$

We call the closure of the set E_2 in the norm corresponding to the scalar product

$$(u^{(1)} \cdot u^{(2)})_{2,0,\omega} = \int_0^\omega [(\mathbf{u}_t^{(1)} \cdot \mathbf{u}_t^{(2)})_1 + (\mathbf{u}^{(1)} \cdot \mathbf{u}^{(2)})_{H_3(\Omega)}]dt$$

the space $X_2\ (0, \omega)$.

Let $E_3\ (0, \omega)$ be a subset of the elements belonging to E_2 which can be represented in the form of the finite sums $\Sigma d_k\ (t)\ \varphi_k$, where $d_k\ (t) \in C^{(2)}\ (0, \omega)$, and satisfy (1.4), and $\varphi_k \in H_3\ (\Omega)$. Following the proofs given in [5], we prove the following lemmas:

Lemma 1. $H_3\ (\Omega)$ is the space $W = \overset{\circ}{W}{}_2^{(1)}\ (\Omega) \times \overset{\circ}{W}{}_2^{(1)}\ (\Omega) \times \overset{\circ}{W}{}_2^{(2)}\ (\Omega)$, and the norms of $H_3\ (\Omega)$ and W are equivalent on $H_3\ (\Omega)$.

Lemma 2. A complete system of vectors $\{\chi_m\ (\chi_{1m}, \chi_{2m}, \chi_{3m})\}$ exists in the space $H_3\ (\Omega)$. The system can be regarded as orthogonal in $H_3\ (\Omega)$ orthonormal in X_1 and such that, if $(\chi_{ip}, \chi_{ip})_1 = 1$, then $(\chi_{jp} \cdot \chi_{jp})_1 = 0,\ i, j = 1, 2, 3,\ p = 1, \ldots, n,\ j \neq i$.

Lemma 3. $X_2\ (0, \omega)$ is a separable Hilbert space and the set $E_3\ (0, \omega)$ is dense everywhere in this space.

Lemma 4. The element $\mathbf{u}_t$ of X_1 and $\mathbf{u}$ the element of $H_3\ (\Omega)$ are functions of $0 \leqslant t \leqslant \omega$ and continuous almost everywhere.

3. Generalized solution and the solvability of the problem.

8) let the following conditions hold:

$$\mathbf{F}\ (t + \omega) = \mathbf{F}\ (t), \ \max \|\mathbf{F}\|_1 < \infty \ \ (\mathbf{F} = (F_1,\ F_2,\ F_3))$$

As we know, the equations of motion of the shell can be expressed, in accordance with the Hamilton-Ostrogradskii principle, in the form

$$\int_0^\omega \Big\{-(\mathbf{u}_t{}^* \cdot \delta\mathbf{u}_t{}^*)_1 + \varepsilon\,(\mathbf{u}_t{}^* \cdot \delta\mathbf{u}^*)_1 + \int_\Omega T_{ij}(\mathbf{u})\, e_{ij}(\delta\mathbf{u}^*)\, d\Omega - (\mathbf{F}^* \cdot \delta\mathbf{u}^*)_1\Big\}\, dt = 0$$

$$\int_0^\omega \Big\{-(u_{3t} \cdot \delta u_{3t})_1 + \varepsilon\,(u_{3t} \cdot \delta u_3)_1 + \int_\omega \Big\{T_{ij}(\mathbf{u})\Big[k_{ij}\delta u_3 + \frac{1}{2}\psi_i(u_3)\psi_j(\delta u_3) + \frac{1}{2}\psi_j(u_3)\,\psi_i\,(\delta u_3)\Big] + M_{ij}(\mathbf{u})\,\varkappa_{ij}(\delta\mathbf{u})\Big\}\, d\Omega - (F_3 \cdot \delta u_3)_1\Big\}\, dt = 0$$

where $\delta\mathbf{u} = (\delta u_1,\ \delta u_2,\ \delta u_3)$ denotes a possible displacement.

Definition. We shall call the generalized ω-periodic solution of the problem (1.1) — (1.3) the vector function $\mathbf{u}\ (t)$ satisfying the conditions

a) $\mathbf{u}\ (t + \omega) = \mathbf{u}_t\ (t),\ \mathbf{u}_t\ (t + \omega) = \mathbf{u}_t\ (t)$,
b) $\max\|\mathbf{u}_t\|_1, \quad \max\|\mathbf{u}\|_{H_3(\Omega)}, \quad \|\mathbf{u}\|_{2,0,\omega}$ are finite

c) the Hamilton-Ostrogradskii equations hold for any $\delta \mathbf{u} \in H_3(\Omega)$ strongly differentiable in t.

Applying the usual variational calculus techniques, we can reduce the problem of finding a generalized, ω-periodic solution, to that of solving the operator equation (1.1) in the space $X_2(0, \omega)$.

We can find an approximate generalized solution using the Bubnov-Galerkin method in the following manner: we construct a sequence $\{\mathbf{u}_n\}$ in the form $\mathbf{u}_n = q_1(t)\chi_1 + q_2(t)\chi_2 + \ldots + q_n(t)\chi_n$ where χ_m are those defined in Lemma 2. The vector $(\mathbf{q}_n(t), \mathbf{q}_{nt}(t)) = (q_1(t), \ldots q_n(t), q_{1t}(t), \ldots q_{nt}(t))$ is determined as a periodic solution of the following nonlinear system of ordinary differential equations:

$$(\overset{*}{\mathbf{u}}_{ntt} \cdot \overset{*}{\chi}_m)_1 + \varepsilon(\overset{*}{\mathbf{u}}_{nt} \cdot \overset{*}{\chi}_m)_1 + \int_\Omega T_{ij}(\mathbf{u}_n)e_{ij}(\chi_m{}^*)\,d\Omega - (\mathbf{F}^* \cdot \chi_m{}^*)_1 = 0 \quad (3.1)$$

$$(u_{3ntt} \cdot \chi_{3m})_1 + \varepsilon(u_{3nt} \cdot \chi_{3m})_1 + \int_\Omega \Big\{T_{ij}(\mathbf{u}_n)\Big[k_{ij}\chi_{3m} + \frac{1}{2}\psi_i(u_{3n})\psi_j(\chi_{3m}) + \quad (3.2)$$

$$\frac{1}{2}\psi_j(u_{3n})\psi_i(\chi_{3m})\Big] + M_{ij}(\mathbf{u}_n)\varkappa_{ij}(\overset{*}{\varkappa}_m)\Big\}\,d\Omega - (F_3 \cdot \chi_{3m})_1 = 0$$

Theorem. Let the conditions (1) — (8) hold and let $\{\chi_m\}$ be the system of vectors defined in Lemma 2. Then

a) the system of equations (3.1), (3.2) has at least one ω-periodic solution for any value of n,

b) the set of approximations $\{\mathbf{u}_n\}$ is weakly compact in $X_2(0, \omega)$,

c) each weak limit $\{\mathbf{u}_n\}$ in $X_2(0, \omega)$ represents a generalized ω-periodic solution of the problem (1.1) — (1.4).

The most important point of the proof of the above theorem consists of confirming that Eqs. (3.1), (3.2) are dissipative [6]. The basic difference between the Bubnov-Galerkin equations in the theory of shallow shells and the corresponding equations in the theory of thin plates [2] reveals itself in the following. Let the positive definite functional of the potential energy of the shell

$$\Phi(\mathbf{u}) = \frac{1}{2}\int_\Omega \{T_{ij}(\mathbf{u})\varepsilon_{ij}(\mathbf{u}) + M_{ij}(\mathbf{u})\varkappa_{ij}(\mathbf{u})\}\,d\Omega$$

be defined on the space $H_3(\Omega)$. The form $\Phi_n \equiv \Phi(\mathbf{u}_n)$ in the theory of plates can be written for $q_m(t)$ as the sum $\Phi_n = \Phi_{2n} + \Phi_{4n}$ of the forms of the second and fourth order. The proof of the dissipative character given in [2] is based on the fact that not only Φ_n is a positive definite form of the variables q_m, but also

$$\sum_{m=1}^{n}\left(\frac{\partial}{\partial q_m}\Phi_n\right)q_m = 2\Phi_{2n} + 4\Phi_{4n}$$

In the theory of shallow shells $\Phi_n = \Phi_{2n} + \Phi_{3n} + \Phi_{4n}$, where Φ_{3n} is a third order functional in q_m. For this reason the form $2\Phi_{2n} + 3\Phi_{3n} + 4\Phi_{4n}$ will not be positive definite with respect to q_m, and this requires an approach different from that used in the theory of shells. To prove the theorem, we multiply (3.1) and (3.2) by q_{mt}, sum over m from 1 to n, and combine the resulting expressions

$$\frac{d}{dt}\left(\frac{1}{2}\|\mathbf{u}_{nt}\|_1^2 + \Phi_n\right) = (\mathbf{F} \cdot \mathbf{u}_{nt})_1 - \varepsilon\|\mathbf{u}_{nt}\|_1^2$$

Next we introduce the function

$$V_n(t) \equiv V_n(q_n(t), q_{nt}(t)) = \frac{1}{2}\|\mathbf{u}_{nt}\|_1^2 + \Phi_n + \alpha \sum_{m=1}^{n} [2(\mathbf{u}_n^* \cdot \chi_m^*)_1 \times$$

$$(\mathbf{u}_{nt}^* \cdot \chi_m^*)_1 + (u_{3n} \cdot \chi_{3m})_1 (u_{3nt} \cdot \chi_{3m})_1] + \beta \sum_{m=1}^{n} [2(\mathbf{u}_n^* \cdot \chi_m^*)_1^2 + (u_{3n} \cdot \chi_{3m})_1^2]$$

and impose the following restrictions on the constants $\alpha > 0$ and $\beta > 0$:

$$1/_2 - \alpha\varepsilon_1^2 > 0, \quad \beta - 1/_2\alpha\varepsilon_1^{-2} > 0$$

Taking the Young's inequality with the constant ε_1^2 into account, we can prove the sufficiency of these inequalities for the positive definiteness of $V_n(t)$. The derivative $V_{nt}(t)$ is found using Eqs. (3.1) and (3.2):

$$V_{nt}(t) = (\mathbf{F} \cdot \mathbf{u}_{nt})_1 - \varepsilon\|\mathbf{u}_{nt}\|_1^2 + \alpha \sum_{m=1}^{n} \Big[2(\mathbf{u}_{nt}^* \cdot \chi_m^*)_1^2 +$$

$$(u_{3nt} \cdot \chi_{3m})_1^2 \Big] + \alpha \sum_{m=1}^{n} \Big[2(\mathbf{u}_n^* \cdot \chi_m^*)_1 \Big\{ -\varepsilon(\mathbf{u}_{nt}^* \cdot \chi_m^*)_1 -$$

$$\int_\Omega T_{ij}(\mathbf{u}_n) e_{ij}(\chi_m^*)\, d\Omega + (\mathbf{F}^* \cdot \chi_m^*)_1 \Big\} + (u_{3n} \cdot \chi_{3m})_1 \Big\{ -\varepsilon \times$$

$$(u_{3nt} \cdot \chi_{3m})_1 - \int_\Omega T_{ij}(\mathbf{u}_n) \Big[k_{ij}\chi_{3m} + \frac{1}{2}\psi_i(u_{3n})\psi_j(\chi_{3m}) +$$

$$\frac{1}{2}\psi_j(u_{3n})\psi_i(\chi_{3m}) \Big] + M_{ij}(\mathbf{u}_n)\varkappa_{ij}(\chi_m) \Big\} d\Omega + (F_3 \cdot \chi_{3m})_1 \Big\} \Big] +$$

$$2\beta \sum_{m=1}^{n} \Big[2(\mathbf{u}_n^* \cdot \chi_m^*)_1 (\mathbf{u}_{nt}^* \cdot \chi_m^*)_1 + (u_{3n} \cdot \chi_{3m})_1 (u_{3nt} \cdot \chi_{3m})_1 \Big]$$

Let now $\alpha\varepsilon = 2\beta$. Then the Young's inequalities with the constants ε_2^2 and ε_3^2 yield

$$V_n(t) \leqslant -a\|\mathbf{u}_{nt}\|_1^2 - \alpha \int_\Omega \{T_{ij}(\mathbf{u}_n)[2e_{ij}(\mathbf{u}_n) - k_{ij}u_{3n}] +$$

$$M_{ij}(\mathbf{u}_n)\varkappa_{ij}(\mathbf{u}_n)\}\, d\Omega + \alpha\varepsilon_3^2\|\mathbf{u}_n\|_1^2 + b\|\mathbf{F}\|_1^2$$

Let $a = \varepsilon - 1/_2\varepsilon_2^2 - 2\alpha > 0$, $b = 1/_2\varepsilon_2^{-2} + \alpha\varepsilon_3^{-2}$ and

$$\Phi_n^\circ \equiv \int_\Omega \{T_{ij}(\mathbf{u}_n)[2e_{ij}(\mathbf{u}_n) - k_{ij}u_{3n}] + M_{ij}(\mathbf{u}_n)\varkappa_{ij}(\mathbf{u}_n)\}\, d\Omega - \varepsilon_3^2\|\mathbf{u}_n\|_1^2$$

L e m m a 5. Let $\|\mathbf{u}\|_{H_3(\Omega)} = R$ be a sphere in the space $H_3(\Omega)$, of sufficiently large radius R and independent of t. If the element $\mathbf{u}_n(t)$ belonging to the space $H_3(\Omega)$ for every fixed $-\infty < t < \infty$ and all n, falls at some $t = t^*$ on a sphere of sufficiently large radius R, then the inequality $\Phi_n^\circ(t^*) \geqslant \delta R$ where $\delta > 0$ is a constant independent of $\mathbf{u}_n(t^*)$, is satisfied.

To prove the lemma we repeat the basic argumentation used in [4] in deriving the a p r i o r i estimate, and take into account the fact that the constants used in [4] are independent of $\mathbf{u}_n(t)$.

From Lemma 5 it follows that the functional Φ_n° increases in the space $H_3(\Omega)$.

therefore a constant $m_3 > 0$ exists independent of $\mathbf{u}_n(t)$ and t, and such that the form $\Phi_n^\circ + m_3$ will be positive definite in $H_3(\Omega)$ (we can take e.g. $m_3 = |\inf\Phi_n^\circ|$ in the sphere $\| u_n \|_{H_3(\Omega)} \leqslant R$). The structure of the forms $V_n(t)$ and $a\|\mathbf{u}_{nt}\|_1^2 + \alpha\Phi^\circ$ implies that constants $m > 0$ and $c > 0$ exist, are independent of $\mathbf{u}_n(t)$ and t and such that

$$a\|\mathbf{u}_{nt}\|_1^2 + \alpha\Phi_n^\circ + c \geqslant mV_n$$

The relations connecting the constants m and c with ε_1^2, ε_2^2, ε_3^2, α and β can be obtained in an explicit form. Taking Lemma 5 into account, we can write

$$V_{nt}(t) \leqslant -mV_n(t) + c + b\|\underline{\mathbf{F}}\|_1^2$$

from which

$$V_n(t) \leqslant V_n(q_n(t_0), q_{nt}(t_0))\, e^{-m(t-t_0)} + (c + b\max_t \|\mathbf{F}\|_1^2)\, m^{-1} \times (1 - e^{-m(t-t_0)})$$
$$\limsup V_n(t) \leqslant (c + b\max_t \|F\|_1^2)\, m^{-1} < \infty \quad \text{when} \quad t \to \infty \tag{3.3}$$

for any finite $V_n(t_0)$, t_0 and all n. This in turn implies that $\max_t \|\mathbf{u}_{nt}\|_1 \leqslant \gamma_1$, $\max_t \|\mathbf{u}_n\|_{H_3(\Omega)} \leqslant \gamma_2$ and $\|\mathbf{u}_n\|_1 \leqslant \gamma_3$, with the constants γ_1, γ_2 and γ_3 finite and independent of $\mathbf{u}_n(t)$, as well as the dissipative character of the system (3.1),(3.2). The latter now implies the existence [6] of at least one subharmonic oscillation of period $k_n\omega$, where k_n is a positive integer dependent on n.

It is proved that the period of oscillations for the problem (1.1) — (1.4) is not $k_n\omega$ but ω, for any n. We introduce the operator $\mathbf{K}(\mathbf{q}_n(0), \mathbf{q}_{nt}(0)) = (\mathbf{q}_n(\omega), \mathbf{q}_{nt}(\omega))$ in the $2n$-dimensional space of the coefficients $(\mathbf{q}_n, \mathbf{q}_{nt})$. The transformation introduced here, is continuous. We consider the domain

$$V_n(\mathbf{q}_n(0), \mathbf{q}_{nt}(0)) < M$$

where the constant M is independent of n and t. The relation (3.3) implies the existence of such a constant. Let M be taken such that

$$M^* = M^{-1}(c + b\max_t \|\mathbf{F}\|_1^2)\, m^{-1} < 1$$

Then, taking (3.3) and Lemma 5 into account, we show that for a sufficiently large M,

$$V_n(\mathbf{q}_n(\omega), \mathbf{q}_{nt}(\omega)) \leqslant V_n(\mathbf{q}_n(0), \mathbf{q}_{nt}(0))\,[e^{-mt} + M^*(1 - e^{-mt})] < V_n(\mathbf{q}_n(0), \mathbf{q}_{nt}(0))$$

and the domain

$$V_n(\mathbf{q}_n(0), \mathbf{q}_{nt}(0)) \leqslant M$$

is a star domain. The Schrauder theorem implies the existence of at least one fixed point of the transformation $\mathbf{K}$. The corresponding solution of the system (3.1),(3.2) will be ω-periodic for any n. From the relation (3.3) and the fact that $X_2(0, \omega)$ is a Hilbert space, follows the weak compactness of the sequence $\{\mathbf{u}_n\}$ in $X_2(0, \omega)$ and the inequality

$$\|\mathbf{u}_0\|_{2,0,\omega} \leqslant \liminf \|\mathbf{u}_n\|_{2,0,\omega} \quad \text{for} \quad n \to \infty$$

where $\mathbf{u}_0$ is a weak limit of the sequence $\{\mathbf{u}_n\}$ in $X_2(0, \omega)$. Using the imbedding theorem [7], we prove, as in [5], that $\mathbf{u}_0$ represents a generalized, ω-periodic solution of the problem (1.1) — (1.4). This proves the theorem.

Notes. 1°. If e.g. we take the constants ε_1^2, ε_2^2, ε_3^2, α and β satisfying the inequalities

$$\alpha < {}^1/_4\,\varepsilon, \quad (2\varepsilon)^{-1} < \varepsilon_1^2 < 2\varepsilon^{-1}, \quad \beta < {}^1/_8\,\varepsilon^2, \quad \varepsilon_2^2 < \varepsilon, \quad \alpha\varepsilon = 2\beta$$

then all restrictions imposed on these constants will be satisfied.

2°. Using the proof of the theorem, we can obtain theorems of existence of an ω-periodic solution of the problem on oscillation of a shell with damping taken into account, under the conditions

$$u_i|_\Gamma = g_i(s, t), \quad i = 1, 2, 3, \quad \left.\frac{\partial u_3}{\partial n}\right|_\Gamma = g_4(s, t)$$

$$g_j(s, t+\omega) = g_j(s, t), \quad g_{jt}(s, t+\omega) = g_{jt}(s, t), \quad j = 1, 2, 3, 4$$

$$g_j(s, t) \in L^\infty(0, \omega), \quad g_{jt}(s, t) \in L^\infty(0, \omega), \quad j = 1, 2, 3, 4$$

$$g_j(s, t) \in H_{1/2}(\Gamma), \quad g_3(s, t) \in H_{3/2}(\Gamma), \quad j = 1, 2, 4$$

where $H_{1/2}(\Gamma)$ and $H_{3/2}(\Gamma)$ are the Sobolev-Slobodetskii spaces.

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PLANE SHORT-WAVE OSCILLATIONS IN THE VICINITY OF THE CONVEX BOUNDARY OF AN ELASTIC BODY

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A. L. POPOV and G. N. CHERNYSHEV
(Moscow)

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We investigate short-wave oscillations of a plane elastic body, concentrated in the vicinity of a smooth convex boundary. We develop an asymptotic process of integrating the dynamic equations of the plane theory of elasticity. We obtain the expressions for the eigenfunctions and natural frequencies of the short-wave oscillations for free and clamped boundaries.

The short-wave (high frequency) oscillations can be studied with the help of various asymptotic methods based, in particular, on the method of rays of geometrical optics. A systematic presentation of the method of rays and its development in the boundary value problems of mathematical physics are given in [1, 2]. The method is used to investigate the asymptotic behavior of the eigen-